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Editors

CONTENTS

Part I: LECTURES

L.D. Paniak and R.J. Szabo

Lectures on two-dimensional noncommutative gauge theory
1. Classical Aspects 1

H. Grosse and R. Wulkenhaar

Regularisation and Renormalisation
of Quantum Field Theories on Noncommutative Spaces 29

M. Maceda and J. Madore

On the Resolution of Space-Time Singularities II 47

S. Waldmann

Deformation Quantization:
Observable Algebras, States and Representation Theory 83

M. Mondragón and G. Zoupanos

Finite unification at all-loops 109

M. Blagojević

Three lectures on Poincaré gauge theory 147

A.T. Filippov

Integrable Low Dimensional Theories Describing
High Dimensional Branes, Black Holes and Cosmologies 173

I. Antoniadis

Mass scales in string theory 197

D. Lüst

Intersecting Brane Worlds and their Effective Interactions 257

E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva

Strings, p -Branes and Dp -Branes With Dynamical Tension 271

Strings, p -Branes and Dp -Branes

With Dynamical Tension

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Abstract

We discuss a new class of brane models (extending both p -brane and Dp -brane cases) where the brane tension appears as an *additional dynamical degree of freedom* instead of being put in by hand as an *ad hoc* dimensionfull scale. Consistency of dynamics naturally involves the appearance of additional higher-rank antisymmetric tensor gauge fields on the world-volume which can couple to charged lower-dimensional branes living on the original Dp -brane world-volume. The dynamical tension has the physical meaning of electric-type field strength of the additional higher-rank world-volume gauge fields. It obeys Maxwell (or Yang-Mills) equations of motion (in the string case $p = 1$) or their higher-rank gauge theory analogues (in the Dp -brane case). This in particular triggers a simple classical mechanism of (“color”) charge confinement.

1 Introduction

The crucial relevance of Dirichlet p -branes (Dp -branes) [1], *i.e.*, $p + 1$ -dimensional extended objects in space-time on which the ends of fundamental open strings can be confined, is largely appreciated and exploited in modern string theory (for reviews of string and brane theories, see [2, 3, 4]). Their importance is primarily due to the following basic properties: providing explicit realization of non-perturbative string dualities, microscopic description of black-hole physics, gauge theory/gravity correspondence, large-radius compactifications of extra dimensions, brane-world scenarios in particle phenomenology, *etc.* .

When building actions in geometrically motivated field theories, which is the class where strings and branes belong, one of the most important ingredients is the consistent generally-covariant integration measure density, *i.e.*, covariant under arbitrary diffeomorphisms (reparametrizations) on the underlying space-time manifold. *A priori* there are no compelling geometric reasons restricting us to the natural choice, which is the standard Riemannian metric density $\sqrt{-g}$ with $g \equiv \det ||g_{\mu\nu}||$. For instance, introducing additional D scalar fields φ^i ($i = 1, \dots, D$ where D is the space-time dimension) we may employ the following alternative non-Riemannian measure density $\Phi(\varphi)$:

$$\Phi(\varphi) \equiv \frac{1}{D!} \varepsilon^{\mu_1 \dots \mu_D} \varepsilon_{i_1 \dots i_D} \partial_{\mu_1} \varphi^{i_1} \dots \partial_{\mu_D} \varphi^{i_D} . \quad (1)$$

Making use of (1) allows to construct a broad class of new models involving Gravity called *Two-Measure Gravitational Models* [5], whose actions are typically of the form:

$$S = \int d^D x \Phi(\varphi) L_1 + \int d^D x \sqrt{-g} L_2 , \quad (2)$$

$$L_{1,2} = e^{\frac{\alpha\phi}{M_P}} \left[-\frac{1}{\kappa} R(g, \Gamma) - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + (\text{Higgs}) + (\text{fermions}) \right] . \quad (3)$$

Here $R(g, \Gamma)$ is the scalar curvature in the first-order formalism (*i.e.*, the connection Γ is independent of the metric), ϕ is the dilaton field, M_P is the Planck mass, *etc.*. Although naively the additional “measure-density” scalars φ^i appear in (2) as pure-gauge degrees of freedom (due to the invariance under arbitrary diffeomorphisms in the φ^i -target space), there is still a remnant – the so called “geometric” field $\chi(x) \equiv \frac{\Phi(\varphi)}{\sqrt{-g}}$, which remains an additional dynamical degree of freedom beyond the standard physical degrees of freedom characteristic to the ordinary gravity models with the standard Riemannian-metric integration measure. The most important property of the “geometric” field $\chi(x)$ is that its dynamics is determined solely through the matter fields locally (*i.e.*, without gravitational interaction). The latter turns out to have a significant impact on the physical properties of the two-measure gravity models which allows them to address various basic problems in cosmology and particle physics phenomenology and provide physically plausible solutions, for instance: (i) the issue of scale invariance and its dynamical breakdown, *i.e.*, spontaneous generation of dimension-full fundamental scales; (ii) cosmological constant problem; (iii) geometric origin of fermionic families.

In what follows we are going to apply the above ideas to the case of string, p -brane and Dp -brane models. To make the exposition self-contained Sections 2 and 3 below review our earlier works [6, 7] by providing a detailed description of the simplest $p = 1$ case - the class of new modified-measure string models. Then in Section 4 we proceed with the extension of our construction to the physically most interesting case of new modified-measure Dp -brane models. Along the way we elaborate on various important properties of the modified-measure string and brane models with dynamical string/brane tension.

2 Bosonic Strings with a Modified World-Sheet Integration Measure

First let us recall the standard Polyakov-type action for the bosonic string [8]:

$$S_{\text{Pol}} = -T \int d^2\sigma \frac{1}{2} \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X). \quad (4)$$

Here $(\sigma^0, \sigma^1) \equiv (\tau, \sigma)$; $a, b = 0, 1$; $\mu, \nu = 0, 1, \dots, D-1$; $G_{\mu\nu}$ denotes the Riemannian metric on the embedding space-time; γ_{ab} is the intrinsic Riemannian metric on the 1 + 1-dimensional string world-sheet and $\gamma = \det ||\gamma_{ab}||$; T indicates the string tension – a dimensionfull scale introduced *ad hoc*. The resulting equations of motion w.r.t. γ^{ab} and X^μ read, respectively:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu \right) G_{\mu\nu}(X) = 0, \quad (5)$$

$$\frac{1}{\sqrt{-\gamma}} \partial_a \left(\sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) + \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma_{\nu\lambda}^\mu = 0, \quad (6)$$

where $\Gamma_{\nu\lambda}^\mu = \frac{1}{2} G^{\mu\kappa} (\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda})$ is the connection for the external metric.

Now, following the same ideas as in the construction of two-measure gravity theories (2)–(3) let us introduce two additional world-sheet scalar fields φ^i ($i = 1, 2$) and replace $\sqrt{-\gamma}$ with a new reparametrization-covariant world-sheet integration measure density $\Phi(\varphi)$ defined solely in terms of φ^i :

$$\Phi(\varphi) \equiv \frac{1}{2} \varepsilon_{ij} \varepsilon^{ab} \partial_a \varphi^i \partial_b \varphi^j = \varepsilon_{ij} \dot{\varphi}^i \partial_\sigma \varphi^j. \quad (7)$$

However, the naively generalized string action:

$$S_1 = -\frac{1}{2} \int d^2\sigma \Phi(\varphi) \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$$

has a problem: the equations of motion w.r.t. γ^{ab} lead to an unacceptable condition $\Phi(\varphi) \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) = 0$, *i.e.*, vanishing of the induced metric on the world-sheet.

To remedy the above situation let us consider topological (total derivative) terms $\int \Omega$ w.r.t. the standard Riemannian world-sheet integration measure. Upon measure replacement $\sqrt{-\gamma} \rightarrow \Phi(\varphi)$, i.e., $\int \Omega \rightarrow \int \Omega \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$, the former are *not any more* topological – they will contribute nontrivially to the equations of motion. For instance:

$$\int d^2\sigma \sqrt{-\gamma} R \rightarrow \int d^2\sigma \Phi(\varphi) R \quad , \quad R = \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} (\partial_a \omega_b - \partial_b \omega_a) \quad , \quad (8)$$

where R is the scalar curvature w.r.t. $D = 2$ spin-connection $\omega_a^{\bar{a}\bar{b}} = \omega_a \varepsilon^{\bar{a}\bar{b}}$ (here \bar{a}, \bar{b} denote tangent space indices). Note that the vector field ω_a behaves as world-sheet Abelian gauge field.

Eq.(8) prompts us to construct the following consistent modified bosonic string action¹:

$$S = - \int d^2\sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A) \right] \quad , \quad (9)$$

where $\Phi(\varphi)$ is given by (7) and $F_{ab}(A) \equiv \partial_a A_b - \partial_b A_a$ is the field-strength of an auxiliary Abelian gauge field A_a . The action (9) is reparametrization-invariant as its ordinary string analogue (4). Furthermore, (9) is invariant under diffeomorphisms in φ -target space supplemented with a special conformal transformation of γ_{ab} :

$$\varphi^i \longrightarrow \varphi'^i = \varphi'^i(\varphi) \quad , \quad \gamma_{ab} \longrightarrow \gamma'_{ab} = J \gamma_{ab} \quad , \quad J \equiv \det \left\| \frac{\partial \varphi'^i}{\partial \varphi^j} \right\| \quad . \quad (10)$$

The latter symmetry, which we will call “ Φ -extended Weyl symmetry”, is the counterpart of the ordinary Weyl conformal symmetry of the standard string action (4).

The equations of motion of the action (9) w.r.t. φ^i :

$$\varepsilon^{ab} \partial_b \varphi^i \partial_a \left(\gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} \right) = 0 \quad (11)$$

¹In ref.[9] another interesting geometric modification of the standard bosonic string model has been proposed, which is based on dynamical world-sheet metric and torsion.

imply (provided $\Phi(\varphi) \neq 0$) :

$$\gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = M \quad (= \text{const}) . \quad (12)$$

The equations of motion w.r.t. γ^{ab} are:

$$T_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \frac{1}{2} \gamma_{ab} \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = 0 . \quad (13)$$

Both Eqs.(12)–(13) yield $M = 0$ and:

$$\left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu \right) G_{\mu\nu}(X) = 0 , \quad (14)$$

which is the same as in standard Polyakov-type formulation (5).

The equations of motion w.r.t. X^μ read:

$$\partial_a \left(\Phi \gamma^{ab} \partial_b X^\mu \right) + \Phi \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma_{\nu\lambda}^\mu = 0 , \quad (15)$$

where again $\Gamma_{\nu\lambda}^\mu$ is the connection corresponding to the external space-time metric $G_{\mu\nu}$ as in the standard string case (6).

Now, let us consider the equations of motion w.r.t. A_a resulting from (9) :

$$\varepsilon^{ab} \partial_b \left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) = 0 . \quad (16)$$

The latter can be integrated to yield a *spontaneously induced* string tension:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = \text{const} \equiv T .$$

Since the modified-measure string model (9) naturally requires the presence of the auxiliary Abelian world-sheet gauge field A_a , we may extend it by introducing a coupling of A_a to some world-sheet charge current j^a :

$$S = - \int d^2\sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A) \right] + \int d^2\sigma A_a j^a . \quad (17)$$

In particular, we may take j^a to be the current of point-like charges on the string, so that in the “static” gauge:

$$\int d^2\sigma A_a j^a = - \sum_i e_i \int d\tau A_0(\tau, \sigma_i), \quad (18)$$

where σ_i ($0 < \sigma_1 < \dots < \sigma_N \leq 2\pi$) are the locations of the charges. Now, instead of (16) the action (17) produces the following A_a -equations of motion:

$$\varepsilon^{ab} \partial_b E + j^a = 0 \quad , \quad E \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}. \quad (19)$$

Note that Eqs.(19) look exactly as $D = 2$ Maxwell equations where the *variable* dynamical string tension $E \equiv \Phi(\varphi)/\sqrt{-\gamma}$ is identified as world-sheet electric field strength, i.e., canonically conjugated momentum w.r.t. A_1 .

The physical meaning of the dynamical string tension as world-sheet electric field strength can be directly verified in the framework of the canonical Hamiltonian treatment of the modified-measure string model (17). Indeed, from the explicit form of the action (17) we find the canonical momenta to be:

$$\pi_i^\varphi = -\varepsilon_{ij} \partial_\sigma \varphi^j \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A) \right], \quad (20)$$

$$\pi_{A_1} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \quad , \quad \mathcal{P}_\mu = -\Phi(\varphi) \left(\gamma^{00} \dot{X}^\nu + \gamma^{01} \partial_\sigma X^\nu \right) G_{\mu\nu}, \quad (21)$$

Using (20)–(21) we obtain the canonical Hamiltonian as a linear combination of first-class constraints only. Part of the latter resemble the constraints in the ordinary string case $\pi_{\gamma^{ab}} = 0$ and

$$\mathcal{T}_\pm \equiv \frac{1}{4} G^{\mu\nu} \left(\frac{\mathcal{P}_\mu}{E} \pm G_{\mu\kappa} \partial_\sigma X^\kappa \right) \left(\frac{\mathcal{P}_\nu}{E} \pm G_{\nu\lambda} \partial_\sigma X^\lambda \right) = 0,$$

where in the last Virasoro constraints the dynamical string tension E appears instead of the *ad hoc* constant tension.

The rest of the Hamiltonian constraints are $\pi_{A_0} = 0$ and

$$\partial_\sigma E - \sum_i e_i \delta(\sigma - \sigma_i) = 0, \quad (22)$$

which is precisely the $D = 2$ ‘‘Gauss law’’ constraint for the dynamical string tension coinciding with the 0-th component of the Maxwell-type Eq.(19). Finally, we have constraints involving only the measure-density fields:

$$\partial_\sigma \varphi^i \pi_i^\varphi = 0 \quad , \quad \frac{\pi_2^\varphi}{\partial_\sigma \varphi^1} = 0. \quad (23)$$

The last two constraints span a closed Poisson-bracket algebra:

$$\left\{ \partial_\sigma \varphi^i \pi_i^\varphi(\sigma), \partial_{\sigma'} \varphi^i \pi_i^\varphi(\sigma') \right\} = 2\partial_\sigma \varphi^i \pi_i^\varphi(\sigma) \partial_\sigma \delta(\sigma - \sigma') + \partial_\sigma \left(\partial_\sigma \varphi^i \pi_i^\varphi \right) \delta(\sigma - \sigma'),$$

(a centerless Virasoro algebra), and:

$$\left\{ \partial_\sigma \varphi^i \pi_i^\varphi(\sigma), \frac{\pi_2^\varphi}{\partial_\sigma \varphi^1}(\sigma') \right\} = -\partial_\sigma \left(\frac{\pi_2^\varphi}{\partial_\sigma \varphi^1} \right) \delta(\sigma - \sigma').$$

Therefore, the constraints (23) imply that the measure-density scalars φ^i are pure-gauge degrees of freedom. The only physical remnant of the latter is the specific combination $\frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ (see first Eq.(21)) – the world-sheet electric field-strength simultaneously playing the role of dynamical string tension.

3 Non-Abelian Generalization

We will now show that it is possible to introduce an alternative form of topological term necessary to make consistent the modified-measure string model which is built in terms of a *non-Abelian* auxiliary world-sheet gauge field.

First, let us notice the following identity in $D = 2$ involving Abelian gauge field A_a :

$$\frac{1}{2\sqrt{-\gamma}} \varepsilon^{ab} F_{ab}(A) = \sqrt{\frac{1}{2} F_{ab}(A) F_{cd}(A) \gamma^{ac} \gamma^{bd}}. \quad (24)$$

This suggests the following proper extension of the modified-measure bosonic string action (9) involving a non-Abelian (e.g., $SU(\mathcal{N})$) auxiliary gauge field A_a (here we take for simplicity flat external metric $G_{\mu\nu} = \eta_{\mu\nu}$) :

$$\begin{aligned} S &= - \int d^2\sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu - \sqrt{\frac{1}{2} \text{Tr}(F_{ab}(A)F_{cd}(A))\gamma^{ac}\gamma^{bd}} \right] \\ &= - \int d^2\sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}} \sqrt{\text{Tr}(F_{01}(A)F_{01}(A))} \right], \end{aligned} \quad (25)$$

where $F_{ab}(A) = \partial_a A_b - \partial_b A_a + i[A_a, A_b]$.

The action (25) is again invariant under the Φ -extended Weyl (conformal) symmetry (10).

Notice that the “square-root” Yang-Mills action (with the regular Riemannian metric integration measure):

$$\int d^2\sigma \sqrt{-\gamma} \sqrt{\frac{1}{2} \text{Tr}(F_{ab}(A)F_{cd}(A))\gamma^{ac}\gamma^{bd}} = \int d^2\sigma \sqrt{\text{Tr}(F_{01}(A)F_{01}(A))} \quad (26)$$

is a “topological” action similarly to the $D = 3$ Chern-Simmons action, i.e., it is *metric-independent*.

Similarly to the Abelian case (17) we can also add a coupling of the auxiliary non-Abelian gauge field A_a to an external “color”-charge world-sheet current j^a :

$$\begin{aligned} S &= - \int d^2\sigma \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}} \sqrt{\text{Tr}(F_{01}(A)F_{01}(A))} \right] \\ &\quad + \int d^2\sigma \text{Tr}(A_a j^a). \end{aligned} \quad (27)$$

In particular, for a current of “color” point-like charges on the world-sheet in the “static” gauge :

$$\int d^2\sigma \text{Tr}(A_a j^a) = - \sum_i \text{Tr} C_i \int d\tau A_0(\tau, \sigma_i), \quad (28)$$

where σ_i ($0 < \sigma_1 < \dots < \sigma_N \leq 2\pi$) are the locations of the charges.

The action (27) produces the following equations of motion w.r.t. φ^i and γ^{ab} , respectively:

$$\frac{1}{2}\gamma^{cd}\partial_c X^\mu\partial_d X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\text{Tr}(F_{01}F_{01})} = M \quad (= \text{const}), \quad (29)$$

$$T_{ab} \equiv \partial_a X^\mu\partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\gamma_{ab}\sqrt{\text{Tr}(F_{01}F_{01})} = 0. \quad (30)$$

As in the Abelian case the above Eqs.(29)–(30) imply $M = 0$ and the Polyakov-type equation (5).

Similarly to the Abelian case (19), the equations of motion of (27) w.r.t. the auxiliary gauge field A_a resemble the $D = 2$ non-Abelian Yang-Mills equations:

$$\varepsilon^{ab}\nabla_b\mathcal{E} + j^a = 0, \quad (31)$$

where:

$$\nabla_a\mathcal{E} \equiv \partial_a\mathcal{E} + i[A_a, \mathcal{E}] \quad , \quad \mathcal{E} \equiv \pi_{A_1} \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}\frac{F_{01}}{\sqrt{\text{Tr}(F_{01}F_{01})}}. \quad (32)$$

Here \mathcal{E} is the non-Abelian electric field-strength – the canonically conjugated momentum π_{A_1} of A_1 , whose norm is the dynamical string tension $T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$.

The equations of motion for the dynamical string tension following from (31) is:

$$\partial_a\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right) + \varepsilon_{ab}\frac{\text{Tr}(F_{01}j^b)}{\sqrt{\text{Tr}(F_{01}^2)}} = 0. \quad (33)$$

In particular, in the absence of external charges ($j^a = 0$): $T \equiv \Phi(\varphi)/\sqrt{-\gamma} = T_0 \equiv \text{const}$

Finally, the X^μ -equations of motion $\partial_a(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu) = 0$ resulting from the action (27) can be rewritten in the conformal gauge $\sqrt{-\gamma}\gamma^{ab} = \eta^{ab}$ as:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\partial^a\partial_a X^\mu - \tilde{j}^a\partial_a X^\mu = 0 \quad , \quad \text{where} \quad \tilde{j}_a \equiv \varepsilon_{ab}\frac{\text{Tr}(F_{01}j^b)}{\sqrt{\text{Tr}(F_{01}^2)}}. \quad (34)$$

For static charges $\tilde{j}_1 = -\sum_i \tilde{e}_i \delta(\sigma - \sigma_i)$:

$$T \equiv \Phi(\varphi)/\sqrt{-\gamma} = T_0 + \sum_i \tilde{e}_i \theta(\sigma - \sigma_i) \quad , \quad \tilde{e}_i \equiv \frac{\text{Tr}(F_{01} C_i)}{\sqrt{\text{Tr}(F_{01}^2)}} \Big|_{\sigma=\sigma_i} ; \quad (35)$$

$$T \partial^a \partial_a X^\mu + \left(\sum_i \tilde{e}_i \delta(\sigma - \sigma_i) \right) \partial_\sigma X^\mu = 0 \quad \rightarrow \quad \begin{cases} \partial^a \partial_a X^\mu = 0 \\ \partial_\sigma X^\mu \Big|_{\sigma=\sigma_i} = 0 \end{cases} . \quad (36)$$

Let us return to the $D = 2$ Yang-Mills-like Eqs.(31) whose 0-th component $\partial_\sigma \mathcal{E} + i[A_1, \mathcal{E}] + j^0 = 0$ is the ‘‘Gauss law’’ constraint for the dynamical string tension ($T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$). For point-like ‘‘color’’ charges and using the gauge $A_1 = 0$ (i.e., $\mathcal{E} \rightarrow \tilde{\mathcal{E}} = G\mathcal{E}G^{-1}$ where $A_1 = -iG^{-1}\partial_\sigma G$), the latter reads:

$$\partial_\sigma \tilde{\mathcal{E}} - \sum_i \tilde{C}_i \delta(\sigma - \sigma_i) = 0 \quad , \quad \tilde{C}_i \equiv GC_i G^{-1} \Big|_{\sigma=\sigma_i} . \quad (37)$$

Let us consider the case of *closed* modified string with positions of the ‘‘color’’ charges at $0 < \sigma_1 < \dots < \sigma_N \leq 2\pi$. Then, integrating the ‘‘Gauss law’’ constraint (37) along the string (at fixed proper time) we obtain:

$$\sum_i \tilde{C}_i = 0 \quad , \quad \tilde{\mathcal{E}}_{i,i+1} = \tilde{\mathcal{E}}_{i-1,i} + \tilde{C}_i , \quad (38)$$

where $\tilde{\mathcal{E}}_{i,i+1} = \tilde{\mathcal{E}}$ in the interval $\sigma_i < \sigma < \sigma_{i+1}$.

The implications of Eqs.(35)–(38) can be summarize as follows:

- Eqs.(35)–(36) tell us that the modified-measure (closed) string with N point-like (‘‘color’’) charges on it ((17) or (27)) is equivalent to N chain-wise connected regular open string segments (stretching from σ_i to σ_{i+1} , $i = 0, 1, \dots, N - 1$) which obey Neumann boundary conditions.
- Each of the above open string segments, with end-points at the charges e_i and e_{i+1} (in the Abelian case) or C_i and C_{i+1} (in the non-Abelian

case), has *different* constant string tension $T_{i,i+1}$ such that $T_{i,i+1} = T_{i-1,i} + \tilde{e}_i$ (the non-Abelian \tilde{e}_i are defined in (35)).

- Eq.(38) shows that the only (classically) admissible configuration of “color” point-like charges coupled to a modified-measure closed bosonic string is the one with *zero* total “color” charge, *i.e.*, the model (27) provides a classical mechanism of “color” charge confinement.

4 Dp -Branes With Dynamical Tension

Our construction from the previous two sections can be extended to the case of higher-dimensional extended objects with dynamical tension embracing both (ordinary) p -branes and Dp -branes. First, let us recall the standard formulation of Dp -branes given in terms of the Dirac-Born-Infeld (DBI) action [4] :

$$S_{DBI} = -T \int d^{p+1}\sigma \left[e^{-\alpha U} \sqrt{-\det \|G_{ab} - \mathcal{F}_{ab}\|} + \frac{\varepsilon^{a_1 \dots a_{p+1}}}{p+1} \mathcal{C}_{a_1 \dots a_{p+1}} \right], \quad (39)$$

with the following short-hand notations:

$$G_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \quad , \quad \mathcal{F}_{ab} \equiv \partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X) - F_{ab}(A) \quad , \quad (40)$$

$$\mathcal{C}_{a_1 \dots a_{p+1}} \equiv \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} C_{\mu_1 \dots \mu_{p+1}}(X) \quad . \quad (41)$$

Here $G_{\mu\nu}(X)$, $U(X)$, $B_{\mu\nu}(X)$, and $C_{\mu_1 \dots \mu_{p+1}}(X)$ are the background metric, the dilaton, 2-form Neveu-Schwarz and $(p+1)$ -form Ramond-Ramond gauge fields, respectively, whereas $F_{ab}(A) = \partial_a A_b - \partial_b A_a$ is the field-strength of the Abelian world-volume gauge field A_a . All world-volume indices take values $a, b = 0, 1, \dots, p$ and $\varepsilon^{a_1 \dots a_{p+1}}$ is the $(p+1)$ -dimensional totally antisymmetric tensor ($\varepsilon^{01 \dots p} = 1$).

Similarly to the string case we now introduce a modified world-volume integration measure density in terms of $p+1$ auxiliary scalar fields φ^i

($i = 1, \dots, p+1$) :

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{i_1 \dots i_{p+1}} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} \varphi^{i_1} \dots \partial_{a_{p+1}} \varphi^{i_{p+1}} , \quad (42)$$

and use it to construct the following new p -brane-type action (coupling to the Ramond-Ramond background field is omitted for simplicity)²:

$$S = - \int d^{p+1} \sigma \Phi(\varphi) \left[e^{-\beta U} \frac{1}{2} \zeta^{ab} (G_{ba} - \mathcal{F}_{ba}) + \frac{1}{\sqrt{-\zeta}} \Omega(\mathcal{A}) \right] + \int d^{p+1} \sigma \mathcal{L}(\mathcal{A}) . \quad (43)$$

Here apart from (40) the following new notations are used. The $(p+1) \times (p+1)$ matrix ζ_{ab} of auxiliary variables is an arbitrary world-volume 2-tensor, ζ^{ab} denotes the corresponding inverse matrix ($\zeta^{ac} \zeta_{cb} = \delta_b^a$) and $\zeta \equiv \det ||\zeta_{ab}||$. The term $\Omega(\mathcal{A})$ indicates a topological density given in terms of some additional auxiliary gauge (or matter) fields \mathcal{A}^I living on the world-volume, where “topological” means:

$$\frac{\partial \Omega}{\partial \mathcal{A}^I} - \partial_a \left(\frac{\partial \Omega}{\partial \partial_a \mathcal{A}^I} \right) = 0 \text{ identically} \quad , \quad \text{i.e. } \delta \Omega(\mathcal{A}) = \partial_a \left(\frac{\partial \Omega}{\partial \partial_a \mathcal{A}^I} \delta \mathcal{A}^I \right) . \quad (44)$$

$\mathcal{L}(\mathcal{A})$ describes possible coupling of the auxiliary fields \mathcal{A}^I to external “currents” on the brane world-volume.

The requirement for $\Omega(\mathcal{A})$ to be a topological density is dictated by the requirement that the new brane action (43) (in the absence of the last gauge/matter term $\int d^{p+1} \sigma \mathcal{L}(\mathcal{A})$) reproduces the standard Dp -brane equations of motion resulting from the DBI action (39) apart from the fact that the Dp -brane tension $T \equiv \Phi(\varphi)/\sqrt{-\zeta}$ becomes now an *additional dynamical degree of freedom* (note that no *ad hoc* dimensionfull tension

²Some time ago a Polyakov-type action classically equivalent to the original DBI-action of the Dp -brane (39) has been proposed in [10] : $S = -T \int d^{p+1} \sigma \sqrt{-\zeta} \left[e^{-\beta U} \frac{1}{2} \zeta^{ab} (G_{ba} - \mathcal{F}_{ba}) - (p-1) \right]$. The advantage of the present more general action (43) is that not only it yields variable dynamical brane tension and naturally introduces additional higher-rank world-volume gauge fields, but also it does not need a “cosmological” term.

factor T has been introduced in (43)). Let us stress that, similarly to the string case, the modified-measure brane model (43) naturally requires (through the necessity to introduce topological density $\Omega(\mathcal{A})$) the existence on the world-volume of an additional (higher-rank tensor) gauge field \mathcal{A} apart from the standard world-volume Abelian vector gauge A_a .

Splitting the auxiliary tensor variable $\zeta^{ab} = \gamma^{ab} + \zeta^{[ab]}$ into symmetric and anti-symmetric parts and setting $\zeta^{[ab]} = 0$, the action (43) reduces to the action of the modified-measure model of ordinary p -branes [7] with Neveu-Schwarz field $B_{\mu\nu}$ and world-volume gauge field A_a disappearing and γ_{ab} assuming the role of world-volume Riemannian metric.

The most obvious example of a topological density $\Omega(\mathcal{A})$ for the additional auxiliary gauge/matter world-volume fields in (43) is:

$$\Omega(\mathcal{A}) = -\frac{\varepsilon^{a_1 \dots a_{p+1}}}{p+1} F_{a_1 \dots a_{p+1}}(\mathcal{A}) \quad , \quad F_{a_1 \dots a_{p+1}}(\mathcal{A}) = (p+1) \partial_{[a_1} \mathcal{A}_{a_2 \dots a_{p+1}]} \quad , \quad (45)$$

where $\mathcal{A}_{a_1 \dots a_p}$ denotes rank p antisymmetric tensor (Abelian) gauge field on the world-volume. Further, as a physically interesting example let us consider the following natural coupling of the auxiliary p -form gauge field:

$$\int d^{p+1} \sigma \mathcal{L}(\mathcal{A}) = \int d^{p+1} \sigma \mathcal{A}_{a_1 \dots a_p} j^{a_1 \dots a_p} \quad (46)$$

to an external world-volume current:

$$j^{a_1 \dots a_p} = \sum_i e_i \int_{\mathcal{B}_i} d^p u \frac{1}{p!} \varepsilon^{\alpha_1 \dots \alpha_p} \frac{\partial \sigma_i^{\alpha_1}}{\partial u^{\alpha_1}} \dots \frac{\partial \sigma_i^{\alpha_p}}{\partial u^{\alpha_p}} \delta^{(p+1)}(\underline{\sigma} - \underline{\sigma}_i(\underline{u})) \quad . \quad (47)$$

Here $j^{a_1 \dots a_p}$ is a current of charged $(p-1)$ -sub-branes \mathcal{B}_i embedded into the original p -brane world-volume via $\sigma^a = \sigma_i^a(\underline{u})$ with parameters $\underline{u} \equiv (u^\alpha)_{\alpha=0, \dots, p-1}$. For simplicity we assume that the \mathcal{B}_i sub-branes do not intersect each other.

With the choices (45) and (46)–(47) the action (43) becomes:

$$S = - \int d^{p+1} \sigma \Phi(\varphi) \left[e^{-\beta U} \frac{1}{2} \zeta^{ab} (G_{ba} - \mathcal{F}_{ba}) - \frac{\varepsilon^{a_1 \dots a_{p+1}}}{(p+1)\sqrt{-\zeta}} F_{a_1 \dots a_{p+1}}(\mathcal{A}) \right]$$

$$- \int d^{p+1} \sigma \mathcal{A}_{a_1 \dots a_p} \varepsilon^{a a_1 \dots a_p} \tilde{j}_a , \quad (48)$$

with $\tilde{j}_a = \sum_i e_i \mathcal{N}_a^{(i)}$, where $\mathcal{N}_a^{(i)}$ is the normal vector w.r.t. world-hyper-surface of the $(p-1)$ -sub-brane \mathcal{B}_i :

$$\mathcal{N}_a^{(i)} \equiv \frac{1}{p!} \varepsilon_{ab_1 \dots b_p} \int_{\mathcal{B}_i} d^p u \frac{1}{p!} \varepsilon^{\alpha_1 \dots \alpha_p} \frac{\partial \sigma_i^{\alpha_1}}{\partial u^{\alpha_1}} \dots \frac{\partial \sigma_i^{\alpha_p}}{\partial u^{\alpha_p}} \delta^{(p+1)}(\underline{\sigma} - \underline{\sigma}_i(\underline{u})) . \quad (49)$$

Let us note that some time ago a modified p -brane model has been proposed in ref.[11] which also contains world-volume p -form gauge field $\mathcal{A}_{a_1 \dots a_p}$. However, the latter model is significantly different from (48) since it is not of Polyakov-type and, moreover, $\mathcal{A}_{a_1 \dots a_p}$ appears there quadratically rather than linearly.

More generally, for $p+1 = rs$ we can have a more general type of topological density entering (43) :

$$\Omega(\mathcal{A}) = \frac{1}{rs} \varepsilon^{a_{11} \dots a_{1r} \dots a_{s1} \dots a_{sr}} F_{a_{11} \dots a_{1r}}(\mathcal{A}) \dots F_{a_{s1} \dots a_{sr}}(\mathcal{A}) \quad (50)$$

with rank $r-1$ (smaller than p) auxiliary world-volume gauge fields.

We may also employ *non-Abelian* auxiliary world-volume gauge fields as in the string case. For instance, when $p=3$ we may take:

$$\Omega(\mathcal{A}) = \frac{1}{4} \varepsilon^{abcd} \text{Tr} (F_{ab}(\mathcal{A}) F_{cd}(\mathcal{A})) \quad (51)$$

or, more generally, for $p+1 = 2q$:

$$\Omega(\mathcal{A}) = \frac{1}{2q} \varepsilon^{a_1 b_1 \dots a_q b_q} \text{Tr} (F_{a_1 b_1}(\mathcal{A}) \dots F_{a_q b_q}(\mathcal{A})) , \quad (52)$$

where $F_{ab}(\mathcal{A}) = \partial_a \mathcal{A}_b - \partial_b \mathcal{A}_a + i[\mathcal{A}_a, \mathcal{A}_b]$.

The modified-measure brane action (48) yields the following equations of motion w.r.t. φ^i and ζ^{ab} :

$$e^{-\beta U} \frac{1}{2} \zeta^{ab} (G_{ba} - \mathcal{F}_{ba}) + \frac{1}{\sqrt{-\zeta}} \Omega(\mathcal{A}) = M \equiv \text{const} , \quad (53)$$

$$e^{-\beta U} (G_{ab} - \mathcal{F}_{ab}) + \zeta_{ab} \frac{1}{\sqrt{-\zeta}} \Omega(\mathcal{A}) = 0 . \quad (54)$$

Both Eqs.(53)–(54) imply:

$$\zeta^{ab} (G_{ba} - \mathcal{F}_{ba}) = 2M \frac{p+1}{p-1} e^{\beta U} \quad , \quad \frac{1}{\sqrt{-\zeta}} \Omega(\mathcal{A}) = -\frac{2M}{p-1} \quad (55)$$

which when substituted in (54) give:

$$G_{ab} - \mathcal{F}_{ab} = \frac{2M}{p-1} e^{\beta U} \zeta_{ab} \quad (56)$$

We now consider the equations of motion of the modified brane action (43) w.r.t. auxiliary (gauge) fields \mathcal{A}^I – these are the equations determining the dynamical brane tension $\mathcal{T} \equiv \Phi(\varphi)/\sqrt{-\zeta}$:

$$\partial_a \left(\frac{\Phi(\varphi)}{\sqrt{-\zeta}} \right) \frac{\partial \Omega}{\partial \partial_a \mathcal{A}^I} + j_I = 0 \quad , \quad (57)$$

where $j_I \equiv \frac{\partial \mathcal{L}}{\partial \mathcal{A}^I} - \partial_a \left(\frac{\partial \mathcal{L}}{\partial \partial_a \mathcal{A}^I} \right)$ is the corresponding “current” coupled to \mathcal{A}^I . Note that the topological nature of $\Omega(\mathcal{A})$ (Eq.(44)) is crucial in the derivation of (57). With the choice (46)–(47), Eq.(57) becomes:

$$\partial_a \left(\frac{\Phi(\varphi)}{\sqrt{-\zeta}} \right) + \tilde{j}_a = 0 \quad , \quad \tilde{j}_a \equiv \sum_i e_i \mathcal{N}_a^{(i)} \quad . \quad (58)$$

It is straightforward to deduce from the action (48) that, similarly to the modified string case (19), the dynamical brane tension is identified as the electric-type field strength, *i.e.*, the canonical momentum corresponding to the p -form gauge field: $\pi_{A_1 \dots p} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\zeta}}$. Accordingly, Eqs.(58) are of the same form as the Maxwell-type equations of motion for the p -form gauge field in $p+1$ dimensions.

In particular, in the absence of coupling of external world-volume currents to the auxiliary (gauge) fields \mathcal{A}^I Eq.(57) (or (58)) imply:

$$\mathcal{T} \equiv \Phi(\varphi)/\sqrt{-\zeta} = C \equiv \text{const} \quad (59)$$

Now, using Eqs.(53) and (56) it is straightforward to show that the modified brane action (43) with $\mathcal{L}(\mathcal{A}) = 0$ classically reduces to the standard

Dp -brane DBI-action (39) :

$$S'_{DBI} = -T' \int d^{p+1} \sigma e^{-\beta' U} \sqrt{-\det \|G_{ab} - \mathcal{F}_{ab}\|} , \quad (60)$$

$$T' \equiv \frac{1}{2} C (2M)^{-\frac{p-1}{2}} (p-1)^{\frac{p+1}{2}} , \quad \beta' \equiv \frac{p+1}{2} \beta , \quad (61)$$

where, however, the Dp -brane tension T' is dynamically generated according to (59) and (61).

More generally, recalling the definition (49) of $\mathcal{N}_a^{(i)}$ we find from (58) that the dynamical brane tension $\mathcal{T} \equiv \Phi(\varphi)/\sqrt{-\zeta}$ is piece-wise constant on the world-volume with jumps when crossing the world-hypersurface of each charged $(p-1)$ -sub-brane \mathcal{B}_i . The corresponding jump being equal to the charge magnitude $\pm e_i$ (the overall sign depending on the direction of crossing w.r.t. the normal $\mathcal{N}_a^{(i)}$).

Finally, let us consider the equations of motion of (43) w.r.t. the Abelian Dp -brane vector gauge field A_a :

$$\partial_b \left(\Phi(\varphi) \zeta^{[ab]} e^{-\beta U} \right) = 0 , \quad (62)$$

and the equations of motion w.r.t. the embedding coordinates X^μ (taking into account Eqs.(62)) :

$$\begin{aligned} & \partial_a \left(\Phi(\varphi) \zeta^{(ab)} \partial_b X^\mu \right) - \frac{1}{2} \Phi(\varphi) G^{\mu\nu} \beta \partial_\nu U \zeta^{[ab]} F_{ab}(A) + \\ & \Phi(\varphi) \partial_a X^\lambda \partial_b X^\nu \left[\zeta^{(ab)} \Gamma_{\lambda\nu}^\mu(\bar{G}) + \frac{1}{2} \zeta^{[ab]} G^{\mu\kappa} \left(\beta \partial_\kappa U B_{\lambda\nu} - \mathcal{H}_{\kappa\lambda\nu}(B) \right) \right] = 0 . \end{aligned} \quad (63)$$

where $\Gamma_{\lambda\nu}^\mu(\bar{G})$ is the connection of the rescaled background metric $\bar{G}_{\mu\nu} = e^{-\beta U} G_{\mu\nu}$:

$$\Gamma_{\lambda\nu}^\mu(\bar{G}) = \frac{1}{2} \bar{G}^{\mu\kappa} \left(\partial_\lambda \bar{G}_{\kappa\nu} + \partial_\nu \bar{G}_{\kappa\lambda} - \partial_\kappa \bar{G}_{\lambda\nu} \right) \quad (64)$$

and $\mathcal{H}_{\kappa\lambda\nu}(\bar{B})$ is the field-strength of the background Neveu-Schwarz two-form gauge field:

$$\mathcal{H}_{\kappa\lambda\nu}(B) = \partial_\kappa B_{\lambda\nu} + \partial_\nu B_{\kappa\lambda} + \partial_\lambda B_{\nu\kappa} \quad (65)$$

Using (58), both Eqs.(62)–(63) can be rewritten in the form:

$$\frac{\Phi(\varphi)}{\sqrt{-\zeta}} \partial_b \tilde{H}^{ab} - \sum_i e_i \mathcal{N}_b^{(i)} \tilde{H}^{ab} = 0 \quad , \quad \tilde{H}^{ab} \equiv \sqrt{-\zeta} \zeta^{[ab]} e^{-\beta U} \quad , \quad (66)$$

and:

$$\frac{\Phi(\varphi)}{\sqrt{-\zeta}} \left[\partial_a \left(\sqrt{-\zeta} \zeta^{(ab)} \partial_b X^\mu \right) + \dots \right] - \sum_i e_i \mathcal{N}_a^{(i)} \sqrt{-\zeta} \zeta^{(ab)} \partial_b X^\mu = 0 \quad . \quad (67)$$

The dots in the l.h.s. of (67) indicate the same expression as in the second line of Eq.(63) with $\Phi(\varphi)$ substituted by $\sqrt{-\zeta}$.

Recalling again the definition (49) of the normal $\mathcal{N}_b^{(i)}$ w.r.t. the charged $(p-1)$ -sub-branes \mathcal{B}_i and the resulting property of the dynamical tension $\mathcal{T} \equiv \Phi(\varphi)/\sqrt{-\zeta}$ being piece-wise constant on the world-volume with jumps when crossing each sub-brane \mathcal{B}_i , we find that Eqs.(66) imply:

$$\partial_b \tilde{H}^{ab} = 0 \quad , \quad \tilde{H}^{a\perp} \Big|_{\mathcal{B}_i} = 0 \quad , \quad (68)$$

where the superscript \perp indicates projection along the normal $N_a^{(i)}$ w.r.t. the world-hypersurface of sub-brane \mathcal{B}_i . The first Eq.(68) can be viewed as Bianchi identity for the field-strength of a new $p-2$ -form world-volume gauge field $B_{a_1 \dots a_{p-2}}$ (which can be viewed as dual variable w.r.t. anti-symmetric part of ζ^{ab}):

$$H_{a_1 \dots a_{p-1}} = (p-1) \partial_{[a_1} B_{a_2 \dots a_{p-1}]} \quad , \quad \tilde{H}^{ab} \equiv \frac{1}{(p-1)!} \varepsilon^{abc_1 \dots c_{p-1}} H_{c_1 \dots c_{p-1}} \quad (69)$$

with \tilde{H}^{ab} as defined in (66). The second Eq.(68) shows that the restrictions of the dual field strength H on each $(p-1)$ -sub-brane \mathcal{B}_i vanish:

$$H_{\alpha_1 \dots \alpha_{p-1}} \Big|_{\mathcal{B}_i} = 0 \quad (70)$$

Here the $\alpha_1, \dots, \alpha_{p-1}$ indicate p -dimensional world-hypersurface tensorial indices on the sub-brane \mathcal{B}_i . Let us note that the objects \tilde{H}^{ab} and $B_{a_1 \dots a_{p-2}}$

already appeared in the Polyakov-type formulation of the standard (non-modified) Dp -brane [10]. The new feature in the present modified-measure Dp -brane model is the condition (70) on the dual field strength.

The same arguments from the previous paragraph applied to (67) imply that X^μ satisfy the standard Dp -brane equations of motion (in the Polyakov-type formulation [10]) :

$$\partial_a \left(\sqrt{-\zeta} \zeta^{(ab)} \partial_b X^\mu \right) - \frac{1}{2} \sqrt{-\zeta} G^{\mu\nu} \beta \partial_\nu U \zeta^{[ab]} F_{ab}(A) + \sqrt{-\zeta} \partial_a X^\lambda \partial_b X^\nu \left[\zeta^{(ab)} \Gamma_{\lambda\nu}^\mu(\bar{G}) + \frac{1}{2} \zeta^{[ab]} G^{\mu\kappa} (\beta \partial_\kappa U B_{\lambda\nu} - \mathcal{H}_{\kappa\lambda\nu}(B)) \right] = 0. \quad (71)$$

together with Neumann boundary conditions on the world-hypersurfaces of each charged $(p-1)$ -sub-brane \mathcal{B}_i :

$$\partial_\perp X^\mu \Big|_{\mathcal{B}_i} = 0 \quad , \quad \partial_\perp \equiv N_a^{(i)} \zeta^{(ab)} \partial_b \quad (72)$$

(recall that the symmetric part $\gamma^{ab} \equiv \zeta^{(ab)}$ plays the role of (inverse) world-volume Riemannian metric).

Let us consider again Eq.(58) and integrate it along arbitrary smooth closed curve Γ on the Dp -brane world-volume which is transversal to (some or all of) the $(p-1)$ -sub-brane \mathcal{B}_i , we obtain the following constraints on the possible sub-brane configurations:

$$\sum_i e_i n_i(\Gamma) = 0 \quad , \quad (73)$$

Here $n_i(\Gamma)$ is the sign-weighted total number of Γ crossing \mathcal{B}_i . Eq.(73) is the brane analog of the ‘‘color’’ charge confinement condition (first Eq.(38)) in the modified-measure string model. In the present $p \geq 2$ -brane case, however, due to the much more complicated topologies of the pertinent world-volumes Eq.(73) may yield various different types of allowed sub-brane configurations.

As a simple illustration, let us consider the simplest non-trivial case $p = 2$ and take the static gauge for the $p = 1$ sub-branes (strings), *i.e.*, the proper times of the charged strings coincides with the proper time of the bulk membrane. The latter means that the fixed-time world-volume of the bulk *closed* membrane is a Riemann surface with some number g of handles and no holes. Further, we will assume the following simple topology of the attached N charged strings \mathcal{B}_i : upon cutting the membrane surface along these attached strings it splits into N *open* membranes \mathcal{M}_i ($i = 1, \dots, N$) with Neumann boundary conditions (cf. (72)), each of which being a Riemann surface with g_i handles and 2 holes (boundaries) formed by the strings \mathcal{B}_{i-1} and \mathcal{B}_i , respectively³. The brane tension of \mathcal{M}_i is a dynamically generated constant T_i where $T_{i+1} = T_i + e_i$. In the present configuration Eq.(73) evidently reduces to the constraint $\sum_i e_i = 0$.

Thus, we conclude that similarly to the string case, modified-measure Dp -brane models describe configurations of charged $(p - 1)$ -branes with charge confinement. Apart from the latter, in general there exist more complicated configurations allowed by the constraint (73), whose properties deserve further study.

5 Conclusions

Finally, let us summarize the main features of the new class of modified-measure string and brane models. The above discussion shows that:

- There exist natural from physical point of view modifications of world-sheet and world-volume integration measures which may significantly affect string and brane dynamics.

³The Euler characteristics of the bulk membrane Riemann surface is $\chi = 2 - 2g$, whereas for the open brane \mathcal{M}_i it is $\chi_i = 2 - 2g_i - 2$, so that $\chi = \sum_i \chi_i$ or, equivalently, $g = 1 + \sum_i g_i$.

- Consistency of dynamics *naturally* requires the introduction of auxiliary world-sheet vector gauge field (in the string case) and higher-rank world-volume antisymmetric tensor gauge fields in the general brane case beyond the standard Dp -brane $U(1)$ vector gauge field.
- The string/brane tension is *not* anymore a constant scale given *ad hoc*, but rather appears as an *additional dynamical degree of freedom* beyond the ordinary string/brane degrees of freedom.
- The dynamical string/brane tension has physical meaning of an electric field strength for the auxiliary world-sheet/world-volume gauge field.
- The dynamical string/brane tension obeys “Gauss law” constraint equation and may be nontrivially variable in the presence of point-like charges (on the string world-sheet) or charged lower-dimensional branes (on the Dp -brane world-volume).
- Modified-measure string/brane models provide simple classical mechanisms for confinement of point-like “color” charges or charged lower-dimensional branes due to variable dynamical tension.

Possible applications of the new class of modified-measure Dp -brane models in the context of modern brane-world theories are currently being studied. As a step in this direction we refer to the recent paper [12] where by employing the modified integration measure density (1) a new conformally invariant brane-world model without (bulk) cosmological constant fine tuning has been constructed.

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References

- [1] Polchinski, J. (1996), “*TASI Lectures on D-Branes*” (hep-th/9611050).
- [2] Ne’eman, Y. and Eizenberg, E. (1995), “*Membranes and Other Extensions*”, World Scientific.
- [3] Green, M., Schwarz, J. and Witten, E. (1987), “*Superstring Theory*”, Vol.1,2, Cambridge Univ. Press;
Polchinski, J. (1998), “*String Theory*”, Vol.1,2, Cambridge Univ. Press.
- [4] Johnson, C. (2002), “*D-Branes*”, Cambridge Univ. Press, and references therein (see also hep-th/0007170).
- [5] Guendelman, E.I. and Kaganovich, A. (1996), *Phys. Rev.* **D53** 7020;
Guendelman, E.I. and Kaganovich, A. (1999), *Phys. Rev.* **D60** 065004;
Guendelman, E.I.(1999), *Mod. Phys. Lett.* **A14** 1043;
Guendelman, E.I. (2000), *Class. Quant. Grav.* **17** 261;
Kaganovich, A. (2001), *Phys. Rev.* **D63** 025022;
Guendelman E.I. and Kaganovich, A. (2002), *Int. J. Mod. Phys.* **A17** 417 (hep-th/0106152);
Guendelman E.I. and Kaganovich, A. (2002), *Mod. Phys. Lett.* **A17** 1227 (hep-th/0110221).
- [6] Guendelman, E.I. (2000), *Class. Quantum Grav.* **17** 3673 (hep-th/0005041);
Guendelman, E.I. (2001), *Phys. Rev.* **D63** 046006 (hep-th/0006079).
- [7] Guendelman, E.I., Kaganovich, A., Nissimov, E. and Pacheva, S. (2002), *Phys. Rev.* **D66** 046003 (hep-th/0203024);
Guendelman, E.I., Kaganovich, A., Nissimov, E. and Pacheva, S. (2002), hep-th/0210062.

- [8] Polyakov, A. (1981), *Phys. Lett.* **103B** 207,211.
- [9] Katanaev, M. and Volovich, I. (1986), *Phys. Lett.* **B175** 413 (hep-th/0209014).
- [10] Abou Zeid, M. and Hull, C. (1997), *Phys. Lett.* **404B** 264.
- [11] Bergshoeff, E., London, L. and Townsend, P.K. (1992), *Class. Quantum Grav.* **9** 2545 (hep-th/9206026).
- [12] Guendelman, E.I. (2003), hep-th/0303048